KUIS 4 KALKULUS 1 (SP 2026)

Materi: Teknik Pengintegralan & Teorema Dasar Kalkulus • **Kode Soal: B**

1. **Soal (30 poin):** Tentukan hasil integral berikut.

$$\int \frac{\tan(\ln(2x))}{5x}\,dx$$

Solusi & Rubrik Penilaian:

Gunakan pemisalan subtitusi $u = \ln(2x)$. **[10 poin]**

Turunan dari u terhadap x adalah:

$$du = \frac{1}{2x}\cdot 2\,dx = \frac{1}{x}\,dx \implies \frac{dx}{x} = du$$

Substitusikan ke dalam integral:

$$\int \frac{\tan(\ln(2x))}{5x}\,dx = \frac{1}{5}\int \tan(u)\,du \text{[10 poin]}$$

Mengingat $\int \tan(u)\,du = \ln|\sec(u)| + C = -\ln|\cos(u)| + C$, maka diperoleh:

$$= \frac{1}{5}\ln|\sec(\ln(2x))| + C \quad \text{atau} \quad -\frac{1}{5}\ln|\cos(\ln(2x))| + C\text{[10 poin]}$$

2. **Soal (30 poin):** Diketahui $F(x) = \int_2^x (t^2+5)\,dt$. Tentukanlah:

   a. $F(2)$
   b. $F'(x)$
   c. $F'(3)$

Solusi & Rubrik Penilaian:

a) **Menentukan $F(2)$:**
Menggunakan sifat integral tentu di mana batas bawah sama dengan batas atas ($\int_a^a f(t)dt = 0$):

$$F(2) = \int_2^2 (t^2+5)\,dt = 0\text{[10 poin]}$$

b) **Menentukan $F'(x)$:**
Berdasarkan Teorema Dasar Kalkulus I (TDK I), yaitu $\frac{d}{dx}\left[\int_a^x f(t)\,dt\right] = f(x)$:

$$F'(x) = \frac{d}{dx}\left[\int_2^x (t^2+5)\,dt\right] = x^2+5\text{[10 poin]}$$

c) **Menentukan $F'(3)$:**
Substitusikan $x = 3$ ke dalam $F'(x)$:

$$F'(3) = 3^2 + 5 = 9 + 5 = 14\text{[10 poin]}$$

3. **Soal (40 poin):** Hitung nilai integral tentu berikut.

$$\int_0^{\pi/6} \frac{\tan^5(x)}{\sec^2(x)}\,dx$$

METODE 1: Substitusi $u = \cos(x)$

Sederhanakan fungsi: $\dfrac{\tan^5(x)}{\sec^2(x)} = \dfrac{\sin^5(x)}{\cos^3(x)} = \dfrac{(1-\cos^2(x))^2}{\cos^3(x)}\sin(x)$:

$$\int_0^{\pi/6} \frac{\tan^5(x)}{\sec^2(x)}\,dx = \int_0^{\pi/6} \frac{(1-\cos^2(x))^2}{\cos^3(x)}\sin(x)\,dx \textbf{[10 poin]}$$

Misalkan $u = \cos(x) \implies du = -\sin(x)\,dx \implies \sin(x)\,dx = -du$.
Perubahan batas: $x = 0 \implies u = 1$ dan $x = \pi/6 \implies u = \frac{\sqrt{3}}{2}$. **[10 poin]**

$$\int_{\sqrt{3}/2}^{1} \left(u^{-3} - \frac{2}{u} + u\right) du = \left[-\frac{1}{2u^2} - 2\ln|u| + \frac{u^2}{2}\right]_{\sqrt{3}/2}^{1} \textbf{[10 poin]}$$

Evaluasi batas atas ($u = 1$) dan bawah ($u = \sqrt{3}/2$):

$$\text{Hasil} = 0 - \left(-\frac{7}{24} + \ln\left(\frac{4}{3}\right)\right) = \boxed{\frac{7}{24} - \ln\left(\frac{4}{3}\right)} \quad \text{atau} \quad \boxed{\frac{7}{24} - \ln(3) + 2\ln(2)} \textbf{[10 poin]}$$

METODE ALTERNATIF: Substitusi $w = \sec(x)$

Ubah integrand menggunakan identitas $\tan^4(x) = (\sec^2(x) - 1)^2$:

$$\frac{\tan^5(x)}{\sec^2(x)} = \frac{\tan^4(x)}{\sec^3(x)} \cdot \sec(x)\tan(x) = \frac{(\sec^2(x)-1)^2}{\sec^3(x)}\sec(x)\tan(x)$$

Misalkan $w = \sec(x) \implies dw = \sec(x)\tan(x)\,dx$.
Perubahan batas: $x = 0 \implies w = \sec(0) = 1$ dan $x = \pi/6 \implies w = \sec(\pi/6) = \frac{2}{\sqrt{3}}$.

$$\int_1^{2/\sqrt{3}} \frac{(w^2-1)^2}{w^3}\,dw = \int_1^{2/\sqrt{3}} \left(w - \frac{2}{w} + w^{-3}\right) dw$$

Lakukan pengintegralan:

$$= \left[\frac{w^2}{2} - 2\ln|w| - \frac{1}{2w^2}\right]_1^{2/\sqrt{3}}$$

Evaluasi batas atas ($w = \frac{2}{\sqrt{3}}$) dan batas bawah ($w = 1$):

$$\text{Batas atas } (w = 2/\sqrt{3}) : \frac{2}{3} - 2\ln\left(\frac{2}{\sqrt{3}}\right) - \frac{3}{8} = \frac{7}{24} - \ln\left(\frac{4}{3}\right)$$

$$\text{Batas bawah } (w = 1) : \frac{1}{2} - 0 - \frac{1}{2} = 0$$

Sehingga diperoleh hasil akhir:

$$\text{Hasil} = \left(\frac{7}{24} - \ln\left(\frac{4}{3}\right)\right) - 0 = \boxed{\frac{7}{24} - \ln\left(\frac{4}{3}\right)} \quad \text{atau} \quad \boxed{\frac{7}{24} - \ln(3) + 2\ln(2)}$$