KUIS 4 KALKULUS 1 (SP 2026)

Materi: Teknik Pengintegralan & Teorema Dasar Kalkulus • **Kode Soal: A**

1. **Soal (30 poin):** Tentukan hasil integral berikut.

$$\int \frac{\tan(\ln(2x))}{3x}\,dx$$

Solusi & Rubrik Penilaian:

Gunakan pemisalan subtitusi $u = \ln(2x)$. **[10 poin]**

Turunan dari u terhadap x adalah:

$$du = \frac{1}{2x} \cdot 2\,dx = \frac{1}{x}\,dx \implies \frac{dx}{x} = du$$

Substitusikan ke dalam integral:

$$\int \frac{\tan(\ln(2x))}{3x}\,dx = \frac{1}{3}\int \tan(u)\,du \textbf{[10 poin]}$$

Mengingat $\int \tan(u)\,du = \ln|\sec(u)| + C = -\ln|\cos(u)| + C$, maka diperoleh:

$$= \frac{1}{3}\ln|\sec(\ln(2x))| + C \quad \text{atau} \quad -\frac{1}{3}\ln|\cos(\ln(2x))| + C\textbf{[10 poin]}$$

2. **Soal (30 poin):** Diketahui $F(x) = \int_1^x (t^2+1)\,dt$. Tentukanlah:

   a. $F(1)$
   b. $F'(x)$
   c. $F'(2)$

Solusi & Rubrik Penilaian:

a) **Menentukan $F(1)$:**
Menggunakan sifat integral tentu di mana batas bawah sama dengan batas atas ($\int_a^a f(t)dt = 0$):

$$F(1) = \int_1^1 (t^2+1)\,dt = 0\textbf{[10 poin]}$$

b) **Menentukan $F'(x)$:**
Berdasarkan Teorema Dasar Kalkulus I (TDK I), yaitu $\frac{d}{dx}\left[\int_a^x f(t)\,dt\right] = f(x)$:

$$F'(x) = \frac{d}{dx}\left[\int_1^x (t^2+1)\,dt\right] = x^2+1\textbf{[10 poin]}$$

c) **Menentukan $F'(2)$:**
Substitusikan $x = 2$ ke dalam $F'(x)$:

$$F'(2) = 2^2 + 1 = 4 + 1 = 5\textbf{[10 poin]}$$

3. Soal (40 poin): Hitung nilai integral tentu berikut.

$$\int_0^{\pi/4} \frac{\sin^5(x)}{\cos^2(x)}\,dx$$

METODE 1: Substitusi $u = \cos(x)$

Ubah pembilang $\sin^5(x) = \sin^4(x)\sin(x) = (1-\cos^2(x))^2\sin(x)$:

$$\int_0^{\pi/4} \frac{\sin^5(x)}{\cos^2(x)}\,dx = \int_0^{\pi/4} \frac{(1-\cos^2(x))^2}{\cos^2(x)}\sin(x)\,dx \textbf{[10 poin]}$$

Misalkan $u = \cos(x) \implies du = -\sin(x)\,dx \implies \sin(x)\,dx = -du$.
Perubahan batas: $x = 0 \implies u = 1$ dan $x = \pi/4 \implies u = \frac{1}{\sqrt{2}}$. **[10 poin]**

$$\int_1^{1/\sqrt{2}} \frac{(1-u^2)^2}{u^2}(-du) = \int_{1/\sqrt{2}}^1 \left(u^{-2} - 2 + u^2\right)\,du = \left[-\frac{1}{u} - 2u + \frac{u^3}{3}\right]_{1/\sqrt{2}}^1 \textbf{[10 poin]}$$

Evaluasi batas atas dan bawah:

$$\text{Hasil} = \left(-\frac{8}{3}\right) - \left(-\frac{23\sqrt{2}}{12}\right) = \boxed{\frac{23\sqrt{2}-32}{12}} \textbf{[10 poin]}$$

METODE ALTERNATIF: Substitusi $w = \sec(x)$

Ubah integrand ke dalam bentuk $\sec(x)$ dan $\tan(x)$:

$$\frac{\sin^5(x)}{\cos^2(x)} = \frac{\sin^4(x)}{\cos^4(x)} \cdot \cos^2(x) \cdot \frac{\sin(x)}{\cos^2(x)} = \frac{(\sec^2(x)-1)^2}{\sec^4(x)}\sec(x)\tan(x)$$

Misalkan $w = \sec(x) \implies dw = \sec(x)\tan(x)\,dx$.
Perubahan batas: $x = 0 \implies w = \sec(0) = 1$ dan $x = \pi/4 \implies w = \sec(\pi/4) = \sqrt{2}$.

$$\int_1^{\sqrt{2}} \frac{(w^2-1)^2}{w^4}\,dw = \int_1^{\sqrt{2}} \left(1 - 2w^{-2} + w^{-4}\right)\,dw$$

Lakukan pengintegralan:

$$= \left[w + \frac{2}{w} - \frac{1}{3w^3}\right]_1^{\sqrt{2}}$$

Evaluasi batas atas ($w = \sqrt{2}$) dan batas bawah ($w = 1$):

$$\text{Batas atas } (w = \sqrt{2}) : \sqrt{2} + \frac{2}{\sqrt{2}} - \frac{1}{3(2\sqrt{2})} = 2\sqrt{2} - \frac{\sqrt{2}}{12} = \frac{23\sqrt{2}}{12}$$

$$\text{Batas bawah } (w = 1) : 1 + 2 - \frac{1}{3} = \frac{8}{3}$$

Sehingga diperoleh hasil akhir:

$$\text{Hasil} = \frac{23\sqrt{2}}{12} - \frac{8}{3} = \boxed{\frac{23\sqrt{2}-32}{12}}$$